PHYS2170 Mathematical Methods 4

Problems Class 2

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1. An electron of charge $q = -e$ and mass m moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ and an electric field $\mathbf{E} = E_0\hat{\mathbf{i}}$ obeys the following equation of motion:

$$m\dot{\mathbf{v}} = q\left[\mathbf{v} \times \mathbf{B} + \mathbf{E}\right] \qquad \text{(initial condition } \mathbf{v}(t=0) = \mathbf{0}\text{).}$$

   (a) For which direction of $\mathbf{B}$ does the particle travel in a straight line?

   (b) For which direction of $\mathbf{B}$ does the motion remain in the $x - y$ plane with $y > 0$?

2. The following equations are only solvable for certain points $\mathbf{r}$. In each case, the solution set is a surface. Identify the surfaces and interpret k, l, m and n. It is best to think geometrically about the vectors wherever possible.
   For example: $|\mathbf{r}| = k$. *In this case the magnitude of* $\mathbf{r}$ *is always the same,* i.e. *the magnitude is* k*. Hence, the solution is all points* $\mathbf{r}$ *that are a distance* k *from the origin: a sphere of radius* k*.*

   (a) $\mathbf{r} \cdot \hat{\mathbf{k}} = l$, where $\hat{\mathbf{k}}$ is the unit vector along the z-axis. *[Draw the unit vector* $\widehat{\boldsymbol{k}}$*...]*

   (b) $\mathbf{r} \cdot \hat{\mathbf{k}} = m|\mathbf{r}|$

   (c) $|\mathbf{r} - (\mathbf{r} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}| = n$ (tricky)!

3. Show that the set of vectors $(1, 0, 1), (1, 1, 0)$, and $(1, -3, 4)$ lie on a line. Give the equation of the line in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is the independent variable parametrizing points on the line.

Draw pictures!!