**PHYS2170 Mathematical Methods 4**

Problems Class 5

1. Recall the definition of the moment of inertia $I$ of a body with uniform mass density $\mu$ about an axis running through the body at point $\mathbf{r}_0$,

$$I = \mu \int dV \; R_\perp^2,$$

where $R_\perp$ is the perpendicular (shortest) distance from a point $\mathbf{r}$ in the bulk to axis of rotation.

Calculate the moment of inertia of a cylinder about an axis through the center and parallel to the cylinder direction. [Recall the volume element in cylindrical coordinates, $dV = \rho \, d\rho \, dz \; d\theta$]. Express your answer in terms of the mass $M$ and radius $R$ of the cylinder.

2. Consider the line integral

$$I = \int_C \left[ \frac{y \, dx}{(x^2 + y^2)^{3/2}} - \frac{x \, dy}{(x^2 + y^2)^{3/2}} \right],$$

where the curve $C$ is the semi-circle (of radius 3) from $(-3, 0)$ to $(3, 0)$, taken along the upper semi-circle.

(a) Recall that the vector differential $d\mathbf{r}$ can be represented as

$$d\mathbf{r} = \begin{cases} dx\widehat{\boldsymbol{i}} + dy\widehat{\boldsymbol{j}} & \text{(Cartesians)} \\ dr\,\widehat{\boldsymbol{r}} + r\,d\phi\widehat{\boldsymbol{\phi}} & \text{(Polars)} \end{cases}$$

Use this information to show that the integrand can be written in the form $\mathbf{V} \cdot d\mathbf{r}$. Hence, extract the components $V_x$ and $V_y$, and then express $\mathbf{V}$ in *both* cartesian and polar coordinates.

(b) Evaluate the integral by converting to polar coordinates [it may help to draw a picture].

[Recall that, in polar coordinates $x = r\cos\phi, y = r\sin\phi, \widehat{\boldsymbol{r}} = \cos\phi\widehat{\boldsymbol{i}} + \sin\phi\widehat{\boldsymbol{j}}$, and $\widehat{\boldsymbol{\phi}} = \frac{\partial \widehat{\boldsymbol{r}}}{\partial \phi}$.]