PHYS2170 Mathematical Methods 4

Problems Class 9

1. Show explicitly that the following functions satisfy the partial differential equation $c\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t}$:
   (a) $f(x,t) = \sinh(x+ct)$
   (b) $f(x,t) = g(x+ct)$, where g is any differentiable function.

2. (a) Write down the general solution to the differential equation

$$\frac{d^2y}{dt^2} = -9y.$$

   (b) Show that $y(t) = \sin(3t-4)$ is also a solution, and write it as a linear combination of the basis functions you chose in (i) [recall that $\sin(a+b) = \sin a\cos b + \sin b\cos a$].

3. (a) Show that a solution to the partial differential equation

$$4\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial t^2}$$

   is $f(x,t) = h(x-2t)$, where $h(z)$ is an arbitrary function.
   (b) Sketch $f(x,t)$ when

$$h(z) = \frac{1}{1+z^2},$$

   at times $t = 0, t = 1$, and $t = 2$.
   (c) Can you think of another type of solution?

4. A gas pressure wave $p(x,t)$ in a tube satisfies the equation $4\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2}$. A wave is set off by a detonation in a region $-0.1 < x < 0.1$, so that:

$$p(x,0) = \begin{cases} 1 & -0.1 < x < 0.1 \\ 0 & \text{everywhere else} \end{cases}$$

$$\left.\frac{\partial p}{\partial t}\right|_{(x,0)} = 0 \quad \text{everywhere.}$$

   (a) Sketch the initial form of $p(x)$.
   (b) By writing $p(x,t) = f(x-ct) + g(x+ct)$ for a suitable value for c, solve the partial differential equation for $p(x,t)$ for all x and for all $t > 0$.
   (c) Sketch the solution.