Please complete (showing your work) and hand in these questions by **1pm, Wednesday 15 February 2005.** Assignments can be handed in at the lecture or to my office (9.84).

1. A domain in the $x-y$ plane is specified by $\mathbf{a} = (0,0,0), \mathbf{b} = (1,2,0), \mathbf{c} = (0,3,0), \mathbf{d} = (-1,1,0)$. Consider the parallelepiped contained between this domain and the same domain translated by the vector $\mathbf{v} = (1,-1,1)$. Compute the the volume V of this parallelepiped.
   (a) In terms of the variables $\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$ (no numbers).
   (b) As a number.

2. Let $\mathbf{V}(\mathbf{r}) = (y\hat{\mathbf{i}} - x\hat{\mathbf{j}})e^{-a(x^2+y^2)^{3/2}}/(x^2+y^2)$, where $\mathbf{r} = (x,y,z)$.
   (a) Write $\mathbf{V}$ in cylindrical coordinates in terms of $\rho, \phi, z, \widehat{\boldsymbol{\rho}}, \widehat{\boldsymbol{\phi}}, \widehat{\boldsymbol{k}}$.
   (b) Sketch $\mathbf{V}(\mathbf{r})$.
   (c) Calculate $\boldsymbol{\nabla} \times \mathbf{V}$ in both cartesian and polar coordinates and verify that they're the same.

3. A certain region in the Dales is specified by the height function $h(x,y) = (x^2+1)/(1+x^2+y^3)$. For this profile,
   (a) Compute the gradient $\boldsymbol{\nabla} h(x,y)$.
   (b) At the point $\mathbf{r}_0 = (1,2)$, what is the slope if you're walking southwest (*i.e.* in the direction of $-\hat{\mathbf{i}} - \hat{\mathbf{j}}$)?
   (c) What is the maximum slope at the point $\mathbf{r}_0$?

4. Compute the Laplacian of
   (a) $\phi_1 = x^2 + 2xy + 3ze^{-(x+y)}$
   (b) $\phi_2 = e^{-5x}\sin 4y \cos 3z$

5. Maxwell's Equations for electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ in free space are (in appropriate units!!)

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1}$$
$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0 \tag{2}$$
$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{3}$$
$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = 0 \tag{4}$$

   (a) By taking the curl of Equations (3-4), show that both $\mathbf{E}$ and $\mathbf{B}$ satisfy the wave equation with speed c; that is, $c^2\nabla^2\mathbf{E} = \partial^2\mathbf{E}/\partial t^2$.
   (b) If $\mathbf{B} = \mathbf{B}_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$ and $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$, where $\mathbf{E}_0$ and $\mathbf{B}_0$ are constant vectors, what relation between the amplitudes $\mathbf{E}_0$ and $\mathbf{B}_0$ (and any other constants) is implied by Equation 3?