PHYS2170 Mathematical Methods 4 (JA Dunningham)
Assignment 3 **Due date: 1pm, Wednesday 26 April 2006**

Please complete these questions, showing your work. Assignments should be handed in at the lecture or to my office (9.84).

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1. Write the Taylor series for
$$f(x) = \frac{1}{1-x}$$
about $x = 0$, to second order.

2. Solve the ODE
$$\frac{dy}{dx} = -9y$$
with boundary condition $y(1) = 12$

3. Solve the ODE
$$\frac{d^2y}{dx^2} = -9y$$
with boundary conditions $y(0) = 0$ and $y'(0) = 12$.

4. Consider the ODE
$$\frac{d^4y}{dx^4} + 4\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} = 0.$$
Calculate the general solution $y(x)$. What is the dimension of the solution space?

5. Solve the PDE
$$3\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t}$$
for $f(x,t)$, with boundary condition $f(x,0) = e^{-x^2}$. Sketch the solution for $t = 0, 1, 2$.

6. (a) Show by substitution that the functions $\sin(x - ct)$ and $\sin(x + ct)$ are solutions to the PDE
$$c^2\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial t^2}.$$
   (b) Hence show that $h(x,t) = \sin(x - ct) + \sin(x + ct)$ is a solution.
   (c) Show that $h(x,t)$ can be written as a *product* of a function of x and another function of t. Describe a physical interpretation of this solution if $h(x,t)$ describes wave motion in space x and time t.