

I expect you to know the following material from *Maths 3* and other previous courses. We will review some of these concepts and revisit others in new and possibly more abstract concepts. If you feel uncomfortable with these concepts it will be worthwhile reviewing them as lectures proceed. I have indicated relevant portions of *Riley* which cover the material, although any other book is fine as well.

1. **Calculus** *Riley Chapters 1-4*

- (a) Special functions: trigonometric, hyperbolic, logarithm, *etc.*
- (b) Differentiation: partial differentiation, chain rule, product rule, *etc.*
- (c) Integration
- (d) Taylor Series

2. **Vectors** (*Riley Chapter 6*)

- (a) The algebra of vectors and scalars: addition and multiplication, commutativity, associativity.
- (b) Scalar (dot) product $\mathbf{a} \cdot \mathbf{b}$ and the concept of *projection* of one vector onto another.
- (c) Vector (cross) product $\mathbf{a} \times \mathbf{b}$.
- (d) Use of vectors to construct equations for lines and planes. [For example. Line: $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$, where \mathbf{a} and \mathbf{b} are vectors and λ parametrizes a point on the line. Plane: $(\mathbf{r} - \mathbf{a}) \cdot \hat{\mathbf{n}} = 0$, where \mathbf{a} is a point (vector) on the plane and $\hat{\mathbf{n}}$ is a unit normal to the plane. There are other definitions as well.]

3. **Integration and coordinate systems** (*Riley 5.1-5.3*)

- (a) Two dimensional polar coordinates, and the transformation between cartesian and polar coordinates (*i.e.* between $\{x, y\}$ and $\{r, \theta\}$).
- (b) Familiarity with multiple integration. *e.g.* solve

$$I = \int_1^5 dx \int_2^6 dy [x^2 y^2 + x \sin(xy)] .$$

- (c) Familiarity with simple line integrals. For example,

$$I = \int_C [(xy) dx + y^2 dy] ,$$

where C is the line $y = x$ from $x = 1$ to $x = 3$.

4. **Vector Calculus**

- (a) Familiarity with and the ability to calculate divergence, gradient, and curl in cartesian coordinates.
- (b) Knowledge of the interpretation of divergence, gradient, and curl.

Vector Calculus Section: Main Goals

1. Understanding

- (a) An appreciation that vectors exist as objects **independent of their parametrization or coordinate system**.
- (b) The appreciation of the difference between these coordinate systems both in terms of **coordinates** and **basis vectors**.
- (c) An understanding of the four basic integral theorems (integral of a derivative, line integral of a gradient, surface integral of the curl of a vector field, and volume integral of a divergence of a vector field) in terms of physical examples.
- (d) An understanding of the basic properties of smoothly varying vector fields.

2. Skills

- (a) The ability to convert vector and scalar fields, as well as integrals, between the most common three dimensional coordinate systems (cartesian coordinates, spherical and cylindrical polar coordinates).
- (b) The ability to apply the integral theorems to simplify algebraic expressions.
- (c) The ability to apply the integral theorems to simplify calculations.
- (d) The ability to construct multidimensional integrals in convenient coordinate systems for the problem at hand, and solve them.
- (e) The ability to simplify and interpret expressions involving vector derivatives (divergence, gradient, and curl) **without** relying on the particular coordinate system.

3. Physical Arenas where these skills will be useful

- (a) Electricity and Magnetism, **E, B**.
- (b) Fluid Mechanics, **v**.
- (c) General Relativity
- (d) Elasticity of Solids
- (e) Superconductivity
- (f) Polymers, Liquid Crystals, and Membranes
- (g)