This question paper consists of 4 printed pages, each of which is identified by the Code Number PHYS10901.

PHYS10901

DEPARTMENT OF PHYSICS AND ASTRONOMY

Second Semester Examinations, May - June 2001

PHYS10901

Electromagnetism 1

Time allowed: 2 hours

Approximate value of some constants

Speed of light in vacuum, c	$2.998 \cdot 10^8$ m s^{-1}
Electron charge, e	$1.602 \cdot 10^{-19}$ C
Electron rest mass, m_e	$9.110 \cdot 10^{-31}$ kg $= 0.511$ MeV c^{-2}
Proton rest mass, m_p	$1.673 \cdot 10^{-27}$ kg $= 938.3$ MeV c^{-2}
Unified atomic mass unit, u	$1.661 \cdot 10^{-27}$ kg $= 931.494$ MeV c^{-2}
Fine structure constant, α	$1/137.036$
Planck's constant h	$6.626 \cdot 10^{-34}$ J s
Avogardos's constant, N_a	$6.022 \cdot 10^{23}$ mol^{-1}
Permeability of free space, μ_0	$4\pi \cdot 10^{-7}$ H m^{-1}
Permittivity of free space, ϵ_0	$8.854 \cdot 10^{-12}$ F m^{-1}
Coulombs's constant, $1/4\pi\epsilon_0$	$8.987 \cdot 10^9$ N m^2 C^{-2}

Some SI prefixes

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-18}	atto	a	10^{-9}	nano	n
10^{-15}	femto	f	10^9	Giga	G
10^{-12}	pico	p	10^{12}	Tera	T

Answer **ALL QUESTIONS IN SECTION A** and **2 QUESTIONS FROM SECTION B.**
A **SUGGESTED** time allocation and an approximate mark is given for each question.
When calculators are used the method of calculation must be made clear.

SECTION A

Answer <u>all</u> questions from this section.
You are advised to spend 60 minutes on this section.
The number of marks per question is indicated on the right.

A1 A charge $q_1 = 1.5 \cdot 10^{-3}$ C is placed at $x = -1.2\mathrm{m}$ on the x-axis. A second charge $q_2 = -0.5 \cdot 10^{-3}$ C is placed at $y = 0.5$ m on the y-axis.

1. Calculate the components (in units of 10^6 N/C) of the electric field vector at the origin.
2. What is the magnitude of the force (in kN) on a third charge of $q_3 = 0.2 \cdot 10^{-3}$ C located at the origin?

[8]

A2 A large plate has a uniform charge density σ. Use Gauß's law to show that the electric field strength is $E = \sigma \,/\, (2 \cdot \epsilon_0)$ [6]

A3 What is the difference in potential energy (in eV) between a free electron and a electron inside a hydrogen atom at an average distance $r = 0.529 \cdot 10^{-10}$ m from the proton? State explicitly which electron has the lower potential energy. [6]

A4 A 100 m long coaxial cable is charged to 1200 V. The cable's capacitance per length is 50 pF per meter.

1. What is the total charge (in μF) stored in the cable?
2. How much energy (in mJ) is stored in the cable?

[8]

A5 Two capacitors of 1 μF and 3 μF are (a) connected in series, (b) connected in parallel and a potential difference of 5 V is applied. Determine in each case (1) the capacity of the system, (2) the charge stored in each capacitor, and (3) the energy of the system. [8]

A6 A particle with an effective mass of $m = 0.5\ \mathrm{GeV/c^2}$ and a velocity close to the speed of light $v \approx c$ follows a circular path moving always perpendicular to a uniform magnetic field of 0.5 Tesla. What is the radius of curvature R in units of cm? [7]

A7 A long straight wire of radius $R = 1$ mm carries a current of $I = 0.5$ A that is uniformly distributed over the cross sectional area of the wire. Apply Ampère's law to find the magnetic field strength (in μT) at a distance $r = 0.5$ mm from the axis inside the wire. [7]

SECTION B

Answer 2 questions from this section.
You are advised to spend 30 minutes on each question.
The number of marks per question is indicated on the right.

B1 Calculate the capacitance C of a cylindrical capacitor. The radius of the inner cylinder is r_1, and the radius of the outer coaxial cylinder is r_2. The length of both cylinders is L.

To answer this question first apply Gauß's law to find the electric field inside the capacitor. Then find the potential V as function of distance r from the center inside the capacitor. Finally substitute the result for the potential into the definition of the capacitance.

Calculate the capacitance in units of pF for a length of 10 cm and a ratio of $(r_2/r_1 = 3\text{cm}/50\mu\text{m})$

[25]

B2 A circuit is composed of two coaxial metallic cylinders of length L and radii a and b. Each cylinder carries a current of I parallel to its axis, but in the opposite direction from the other cylinder.

Show that the self inductance is

$$L = \frac{\mu_0 L}{2\pi} \ln\left(\frac{a}{b}\right)$$

To show this start by applying Ampères law to get the magnetic field strength as function of distance for $a < r < b$. Then find the magnetic flux by integration over an area which lies in a plane through the common axis and is defined by the intersection with the cylinder surfaces. [25]

B3 Show that the energy E required to build a spherical charge Q distributed evenly throughout the volume of a sphere of radius R is

$$E = \frac{3}{5}\,\frac{Q^2}{4\pi\epsilon_0\, R}$$

To do this first formulate an equation for the charge q of a sphere of radius r that has not yet reached its full charge Q and final radius R. Then use this equation to find the increase dq in charge for a change in radius dr. Write down the potential V as function of radius r by eliminating q. Formulate the energy dE needed to add a charge dq to the sphere. Express the energy dE as function of r and of the increase in radius dr. Finally integrate over dr to find the energy.

Estimate the electric potential energy (in MeV) stored in an uranium nucleus with in total $A = 235$ nucleons of which $Z = 92$ are protons. The radius of an atomic nucleus is approximately $R \approx 10^{-15}\,\text{m} \cdot A^{1/3}$.

[25]

B4 Find the magnetic field (in μT) at the center of a square current loop of side $L = 0.5$ m and carrying a current of 1.5 A.

Start by showing in a diagram how each side contributes to the magnetic field at the center. Find the magnetic field by integration over current elements $I \cdot dl$. To later integrate over θ it is convenient to express a line element as $dl = (r^2/y)\,d\theta$ where y is the shortest distance from the center to the current loop, r is the shortest distance between the center and a current element, and θ the angle between the lines representing y and r. [25]